II.—THE PRINCIPLES OF DEMONSTRATIVE INDUCTION (I.)

BY C. D. BROAD.

SOME years ago I wrote two articles in MIND on *Induction and Probability*, and, more recently, in my presidential address to the Aristotelian Society I tried to state as fully and clearly as I could the present position of the logical theory of what Mr. Johnson calls "Problematic Induction." In the present paper I propose to do the same for what he calls "Demonstrative Induction." In the former undertaking I was greatly indebted to Mr. Keynes, and in this I am even more indebted to Mr. Johnson. All my raw material is contained in his work on *Logic*, and I can claim no more than to have beaten it into a more coherent shape than that in which he left it. I think that my approach to the subject by way of the notions of Necessary and Sufficient Conditions has certain advantages, and that I have been able to make some extensions of the theory. This must be my excuse for publishing a rather long and tedious essay on a somewhat hackneyed subject which has been treated so fully and so recently by a logician of Mr. Johnson's eminence.

1. DEFINITION OF "DEMONSTRATIVE INDUCTION."—A demonstrative induction is a mixed hypothetical syllogism of the form *Modus Ponendo Ponens* (*i.e.*, if p then q, But p, Therefore q), in which the premises are of a certain form. The major premise must be either of the form (*a*) If *this* S is P then *all* S is P, or (*b*) If *at least one* S is P then *all* S is P. In the first case the minor premise must be of the form *This* (same) S is P. In the second case the minor premise must be either of the form *This* S is P, or of the form *At least one* S is P. (It is of course obvious that the former implies the latter, whilst the latter does not imply the former.) The conclusion is always of the form *All* S is P.

We can sum this up in words as follows. The major premise must be a hypothetical proposition, in which the consequent is a universal categorical, and the antecedent is either a singular or a particular categorical of the same quality and with the same subject and predicate terms as the consequent. The minor premise must be the antecedent in the major if this be *singular*. If the antecedent in the major be *particular* the minor premise may be either this antecedent or may be a singular proposition with the same subject and predicate terms and the same quality as the antecedent. The conclusion is always the consequent in the major premise.

In the notation of *Principia Mathematica* the three forms of demonstrative induction may be symbolised as follows :—

$$\begin{array}{ll} \phi a \,.\, \psi a : \supset : \phi x \supset_x \psi x & \\ \phi a \,.\, \psi a & \text{(I.)} \\ \therefore \phi x \supset_x \psi x. & \end{array}$$

$$\begin{array}{ll} (\exists x) \,.\, \phi x \,.\, \psi x : \supset : \phi x \supset_x \psi x & \\ \phi a \,.\, \psi a & \text{(II}a.\text{)} \\ \therefore \phi x \supset_x \psi x. & \end{array}$$

$$\begin{array}{ll} (\exists x) \,.\, \phi x \,.\, \psi x : \supset : \phi x \supset_x \psi x & \\ (\exists x) \,.\, \phi x \,.\, \psi x & \text{(II}b.\text{)} \\ \therefore \phi x \supset_x \psi x. & \end{array}$$

An example would be : "If someone who sleeps in the dormitory has measles, then everyone who sleeps in the dormitory will have measles. But Jones sleeps in the dormitory and has measles. (Or, alternatively, Someone who sleeps in the dormitory has measles.) Therefore everyone who sleeps in the dormitory will have measles." This illustrates II*a*. and II*b*. The following would illustrate I.: "If the gas Hydrogen can be liquefied, then every gas can be liquefied. But the gas Hydrogen can be liquefied. Therefore every gas can be liquefied." I think it is worth while to note that when we use a major premise of this form we are generally taking an extreme instance (*e.g.*, Hydrogen, because it is the lightest and most "gassy" of all gases), and then arguing that if *even* this has a certain property all other members of the same class will *a fortiori* have it. Another example would be the premise: "If the philosopher X can detect no fallacy in this argument no philosopher will be able to detect a fallacy in it." We might be prepared to accept this premise on the grounds of the extreme acuteness of X. But we certainly should not be prepared to accept the premise: "If some philosopher or other can detect no fallacy in this argument then no philosopher will be able to detect a fallacy in it." For the philosopher Y might well rush in where X would fear to tread.

In all cases that we are likely to have to consider, the major premise of a demonstrative induction rests ultimately on a problematic induction. In all such cases it will only have a certain degree of probability. Consequently, although the conclusions of demonstrative inductions do follow of necessity from their premises, they are only probable, because one at least of the premises is only probable. It may happen that both the premises are only probable. Take, *e.g.*, Mr. Johnson's example about the atomic weight of Argon. The ultimate major premise is no doubt the proposition that if some sample of a chemical element has a certain atomic weight then all samples of that element will have that atomic weight. This is a problematic induction from an enormous number of chemical facts, and is only probable. (In fact, owing to the existence of Isotopes, it is not unconditionally true.) But one would also need the premise that Argon is a chemical element. This is again a problematic induction from a large number of chemical facts. And it is only probable.

The argument about Argon, when fully stated, would take the following form: (i) If some sample of a chemical element has a certain atomic weight, then all samples of that element will have that atomic weight. But Argon is a chemical element. Therefore if some sample of Argon has a certain atomic weight W all samples of Argon will have the atomic weight W. (This is an ordinary syllogism.) (ii) Therefore if some specimen of Argon has the atomic weight 40 all specimens of Argon will have the atomic weight 40. (This is a conclusion drawn by the Applicative Principle.) (iii) This specimen of Argon has atomic weight 40. Therefore all specimens of Argon will have atomic weight 40. (This is the demonstrative induction.) The empirical premises are three, *viz.*, the original generalisation about chemical elements, the proposition that Argon is an element, and the proposition that the atomic weight of this specimen of Argon is 40.

Now much the most important major premises for demonstrative inductions are provided by causal laws. It will therefore be necessary for us to consider next the question of Causal Laws.

2. Causal Laws.—The word "cause" is used very ambiguously in ordinary life and even in science. Sometimes it means a necessary, but it may be insufficient, condition. (*e.g.*, "sparks cause fires"). Sometimes it means a sufficient, but it may be more than sufficient, condition or set of conditions (*e.g.*, "Falling from a cliff causes concussion"). Sometimes it means a set of conditions which are severally necessary and jointly sufficient. But, in any interpretation, it involves one or both of the notions

of "necessary" and "sufficient" condition. It is therefore essential to begin by defining these notions and proving the most important general propositions that are true about them.

There is one other preliminary remark to be made. There are two different types of causal law, a cruder and a more advanced. The cruder type merely asserts connexions between *determinable* characteristics. It just says that whenever such and such determinable characteristics are present such and such another determinable characteristic will be present. An example would be the law that cloven-footed animals chew the cud, or that rise of temperature causes bodies to expand. I shall call such laws "*Laws of Conjunction of Determinables.*" The more advanced type of law considers the determinate values of conjoined determinates. It gives a formula from which the determinate values of the effect-determinables can be calculated for every possible set of determinate values of the cause-determinables. An example would be the law for gases that P = RT/V. I will call such laws "*Laws of Correlated Variation of Determinates.*" In the early stages of any science the laws are of the first kind, and in many sciences they have never got beyond this stage, *e.g.*, in biology and psychology. But the ideal of every science is to advance from laws of the first kind to laws of the second kind. Now Mill's Methods of Agreement, Difference, and the Joint Method, are wholly concerned with the establishment of laws of conjunction of determinables. His Method of Concomitant Variations *ought* to have been concerned with the establishment of laws of correlated variation of determinates. But, since he talks of it as simply a weaker form of the Method of Difference, which we have to put up with when circumstances will not allow us to use that method, it is plain that he did not view it in this light. On the other hand, Mr. Johnson's Methods are definitely concerned with laws of correlated variation. They presuppose that laws of conjunction of determinables have already been established.

The order which I shall follow henceforth is this: (i) I shall deal with the notion of necessary and sufficient conditions wholly in terms of determinables. I shall then state Mill's Methods in strict logical form and show what each of them would really prove. (ii) I shall then pass to the notion of correlated variation of determinates, and explain Mr. Johnson's methods.

3. Necessary and Sufficient Conditions.

(i) *Notation.*—The letters E, and C_1, C_2, etc., are to stand for determinable characteristics. I shall use C's to denote determining factors and E's to denote determined factors.

(ii) *Definitions.*—"C is a *sufficient condition* ('S.C.') of E" means "Everything that has C has E" (1).

"C is a *necessary condition* ('N.C.') of E" means "Everything that has E has C" (2).

"$C_1 \ldots C_n$ is a *smallest sufficient condition* ('S.S.C.') of E" means that "$C_1 \ldots C_n$ is a S.C. of E, and no selection of factors from $C_1 \ldots C_n$ is a S.C. of E" (3).

"$C_1 \ldots C_m$ is a *greatest necessary condition* ('G.N.C.') of E" means that "C_1 and C_2 and ... C_m are each a N.C. of E, but nothing outside this set is a N.C. of E" (4).

"$C_1 \ldots C_n$ are *severally necessary and jointly sufficient* to produce E" means that "$C_1 \ldots C_n$ is both a S.S.C. and a G.N.C. of E" (5).

(*N.B.*—I have represented the effect-determinable by the single letter E. This is not meant to imply that it really consists of a single determinable characteristic. In general, it will be complex, like the cause-determinable, and will be of the form $E_1 \ldots E_m$. But in the propositions which I am going to prove in the next few pages the complexity of the effect-determinable is irrelevant, and so it is harmless and convenient to denote it by a single letter. Later on I shall prove a few propositions in which it is necessary to take explicit account of its internal complexity.)

(iii) *Postulates.*—(1) It is assumed that all the C-factors are capable of independent presence or absence. This involves (*a*) that none of them is either a conjunction or alternation of any of the others. (*E.g.*, C_3 must not be the conjunctive characteristic C_1-and-C_2. Nor may it be the alternative characteristic C_1-or-C_2.) Again (*b*) no two of them must be related as red is to colour (for then the first could not occur without the second), or as red is to green (for then the two could not occur together). It is also necessary to assume that all combinations are *causally* possible. For otherwise we might have the two causal laws "Everything that has C_1C_2 has C_3" and "Everything that has C_3 has E." In that case both C_1C_2 and C_3 would have to be counted as S.C.'s of E, since the law "Everything that has C_1C_2 has E" would follow as a logical consequence of these two other laws. This would obviously be inconvenient; we want to confine our attention to *ultimate* causal laws. Our present postulate may be summed up in the proposition that, if there be n cause-factors, it is assumed that all the $2^n - 1$ possible selections (including all taken together) are both logically and causally possible. This may be called the "Postulate of *Conjunctive Independence*."

(2) It is further assumed that every occurrence of any deter-

minable characteristic E has a S.S.C. This means that, whenever the characteristic E occurs, there is some set of characteristics (not necessarily the same in each case) such that the presence of this set in any substance carries with it the presence of E, whilst the presence of any selection from this set is consistent with the absence of E. This is the form which the Law of Universal Causation takes for the present purpose. We will call it "The Postulate of *Smallest Sufficient Conditions*."

(iv) *Propositions.*—(1) "If C be a S.C. of E, then any set of conditions which contains C as a factor will also be a S.C. of E."

Let such a set of conditions be denoted by CX.

Then : (*a*) All that has CX has C.

(*b*) All that has C has E. (Df. 1.)

Therefore all that has CX has E.

Therefore CX is a S.C. of E. (Df. 1.) Q.E.D.

(2) "If $C_1 \ldots C_m$ be a N.C. of E, then any set of conditions contained in $C_1 \ldots C_m$ will also be a N.C. of E."

Consider, *e.g.*, the selection C_1C_2.

Then : (*a*) All that has $C_1 \ldots C_m$ has C_1C_2.

(*b*) All that has E has $C_1 \ldots C_m$. (Df. 2.)

Therefore all that has E has C_1C_2.

Therefore C_1C_2 is a N.C. of E. (Df. 2.) Q.E.D.

(3) "Any S.C. of E must contain all the N.C.'s of E."

Let X be a S.C. of E, and let Y be a N.C. of E.

Then : (*a*) All that has X has E. (Df. 1.)

(*b*) All that has E has Y. (Df. 2.)

Therefore all that has X has Y.

But all the C's are capable of independent presence or absence. (Postulate 1.) Hence this can be true only if X be of the form YZ.

Therefore any S.C. of E must contain as factors every N.C. of E, if E has any N.C.'s. Q.E.D.

(4) "E cannot have more than one G.N.C."

Let $C_1 \ldots C_m$ be a G.N.C. of E. Then this set (*a*) contains *nothing but* N.C.'s of E (Prop. 2); and (*b*) contains *all* the N.C.'s of E. (Df. 4.)

Now any alternative set must either (*a*) contain some factor which is not contained in this one; or (*b*) contain no factor which is not contained in this one. In the first case it will contain some factors which are not N.C.'s of E. Therefore such a set could not be a G.N.C. of E. In the second case this set either coincides with $C_1 \ldots C_m$ or is a selection from $C_1 \ldots C_m$.

On the first alternative it does not differ from $C_1 \ldots C_m$. On the second alternative it does not contain *all* the N.C.'s of E.

Therefore it could not be a G.N.C. of E.

Therefore E cannot have more than one G.N.C. Q.E.D.

(5) " E can have a plurality of S.S.C.'s. These may be either entirely independent of each other, or they may partially overlap; but one cannot be wholly contained in the other."

Take, *e.g.*, C_1C_2 and $C_3C_4C_5$.

To say that C_1C_2 is a S.S.C. of E is to say that everything which has C_1C_2 has E; whilst C_1 can occur without E, and C_2 can occur without E. (Df. 3.)

To say that $C_3C_4C_5$ is a S.S.C. of E is to say that everything which has $C_3C_4C_5$ has E; whilst C_3C_4 can occur without E, and C_4C_5 can occur without E, and C_5C_3 can occur without E. (Df. 3.)

It is evident that the two sets of statements are logically independent of each other, and can both be true.

Now take C_1C_2 and C_2C_3.

We have already stated what is meant by saying that C_1C_2 is a S.S.C. of E. To say that C_2C_3 is a S.S.C. of E means that everything which has C_2C_3 has E; whilst C_2 can occur without E, and C_3 can occur without E. If the two sets of statements be compared it will be seen that they are quite compatible with each other.

But it would be impossible, *e.g.*, for C_1C_2 and C_1 to be both of them S.S.C.'s of E. For, if C_1C_2 were a S.S.C., it would follow from Df. 3 that C_1 would not be a S.C. at all. Q.E.D.

(6) " Any factor which is common to all the S.S.C.'s of E is a N.C. of E."

Let S_1, S_2, and S_3 be *all* the S.S.C.'s of E. And let C be a factor common to all of them.

Since every occurrence of E has a S.S.C. (Postulate 2), everything that has E has either S_1 or S_2 or S_3.

But everything that has S_1 has C, and everything that has S_2 has C, and everything that has S_3 has C.

Therefore everything that has E has C.

Therefore C is a N.C. of E. (Df. 2.) Q.E.D.

(7) " If E has *only one* S.S.C., it has also a G.N.C., and these two are identical. And so this set is severally necessary and jointly sufficient to produce E."

By Prop. 4 there cannot be more than one G.N.C. of E.

By Prop. 3 the S.S.C. of E must contain the G.N.C. of E.

By Prop. 6 any factor that is common to all the S.S.C.'s of E must be a N.C. of E. Now, since in the present case there is *only one* S.S.C. of E, *every* factor in it is common to all the S.S.C.'s of E.

Therefore every factor in the S.S.C. of E is a N.C. of E.

But we have already shown that every N.C. of E must be a factor in the S.S.C. of E.

Therefore the S.S.C. and the G.N.C. of E coincide.

Therefore this set of factors is severally necessary and jointly sufficient to produce E. Q.E.D.

(8) "If C be a S.C. of E_1 and also a S.C. of E_2; then it will also be a S.C. of E_1E_2. And the converse of this holds also."

The hypothesis is equivalent to the two propositions :—

All that has C has E_1 ; and
All that has C has E_2. (Df. 1.)

Now these are together equivalent to the proposition: "All that has C has E_1E_2." And this is equivalent to the proposition: "C is a S.C. of E_1E_2." (Df. 1.) Q.E.D.

(9) "If C be a N.C. of either E_1 or E_2, then it is a N.C. of E_1E_2."

If C be a N.C. of E_1 it follows from Df. 2 that all that has E_1 has C.

But all that has E_1E_2 has E_1.

Therefore all that has E_1E_2 has C.

Therefore, by Df. 2, C is a N.C. of E_1E_2.

In exactly the same way it can be shown that, if C be a N.C. of E_2, it will be a N.C. of E_1E_2.

Therefore, if C be a N.C. either of E_1 or of E_2, it will be a N.C. of E_1E_2. Q.E.D.

(10) "The converse of (9) is false. It is possible for C to be a N.C. of E_1E_2 without its being a N.C. of E_1 or a N.C. of E_2."

If C be a N.C. of E_1E_2, then all that has E_1E_2 has C. (Df. 2.)

But this is quite compatible with there being some things which have E_1 without having C, or with there being some things which have E_2 without having C. (*E.g.*, all things that are black and human have woolly hair. But there are black things and there are human things which do not have woolly hair.)

So the truth of the proposition that C is a N.C. of E_1E_2 is compatible with the falsity of either or both the propositions that C is a N.C. of E_1 and that C is a N.C. of E_2. Q.E.D.

(11) "If C_1C_2 be a S.C. of *each* of the effect-factors E_1, E_2, . . . E_n, and if it be a S.S.C. of *at least one* of them, then it will be a S.S.C. of the complex effect $E_1 \dots E_n$."

From Prop. 8 it follows at once that C_1C_2 will be a S.C. of $E_1 \ldots E_n$. It is therefore only necessary to show that it will be a *S*.S.C.

Let us suppose, *e.g.*, that C_1C_2 is a S.S.C. of the factor E_1. Then, from Df. 3, it follows that C_1 is not a S.C. of E_1 and that C_2 is not a S.C. of E_1.

Now suppose, if possible, that C_1C_2 is not a S.S.C. of $E_1 \ldots E_n$. We know that it is a S.C. of $E_1 \ldots E_n$. If it be not a S.S.C., then either C_1 or C_2 must be a S.C. of $E_1 \ldots E_n$. (Df. 3.) But, if so, then either C_1 or C_2 must be a S.C. of E_1. (Prop. 8.) But we have seen above that neither C_1 nor C_2 can be a S.C. of E_1.

Hence the supposition that C_1C_2 is not a S.S.C. of $E_1 \ldots E_n$ is impossible. Q.E.D.

(12) "The converse of (11) is false. If C_1C_2 be a S.S.C. of $E_1 \ldots E_n$, it will indeed be a S.C. of each of the factors; but it need not be a S.S.C. of any of the factors."

This is obvious. *E.g.*, C_1 might be sufficient to produce E_1, though nothing less than C_1C_2 was sufficient to produce $E_1 \ldots E_n$.

4. THE POPULAR-SCIENTIFIC NOTION OF "CAUSE" AND "EFFECT."—The notions which we have been defining and discussing above are those which emerge from the looser notions of "cause" and "effect," which are current in daily life and the sciences, when we try to make them precise and susceptible of logical manipulation. There are, however, certain points which must be cleared up before the exact relation between the logical and the popular-scientific notions can be seen.

(i) *The Time-factor*.—It might well be objected that the notion of temporal succession is an essential factor in the common view of cause and effect, and that this has disappeared in our account of necessary and sufficient conditions. The effect is conceived as something that begins at the same moment as the cause ends. And without this temporal distinction it would be impossible to distinguish effect from cause. All this is perfectly true, and it would be of great importance to make it quite explicit if one were dealing with the metaphysics, as distinct from the mere logical manipulation, of causation. But for the present purpose it may be met by the following remark about our notation. We must think of some at least of our C's as being really of the complex form "being characterised by $\mathfrak{C}$ up to the moment *t*." and of some of our E's as being really of the complex form "beginning to be characterised by $\mathfrak{E}$ at the moment *t*."

(ii) *Transeunt Causation*.—A second highly plausible objection would be the following. In our exposition of necessary and sufficient conditions we have always talked of a single continuant, and have supposed that the effect-characteristics and the cause-characteristics occur in the same continuant. But in fact most causation is transeunt, *i.e.*, the cause-event takes place in one continuant and the effect-event in another. This, again, is perfectly true, and very important in any attempt at an analysis of causation for metaphysical purposes. The usual kind of causal law does in fact take roughly the following form: "If a continuant having the properties P is in the state S_1 at a moment *t* and it then comes into the relation R to a continuant which has the properties P′ and is in the state S'_1, the former continuant will begin to be in the state S_2 and the latter in the state S'_2." *E.g.*, "If a hard massive body moving in a certain direction and with a certain velocity at a certain moment comes at that moment into contact with a soft inelastic body at rest, the motion of the former body will begin to change and a dint will develope in the latter body."

For mere purposes of logical manipulation, however, all this can be symbolised as changes in the characteristics of the first continuant. We shall have to remember that some of our C's and some of our E's stand for relational properties of a very complex kind, involving relations to other continuants. Thus, in the example one of our C's will be the characteristic of "Coming into contact at *t* with a soft inelastic resting body." And one of our E's will be the characteristic of "Having been in contact at *t* with the same body beginning to develope a dint." All this is purely a matter of verbal and notational convenience. It has no philosophical significance. But it is harmless so long as we remember that our innocent-looking C's and E's stand, not just for simple qualities, but for extremely complex relational properties of the various kinds described above.

(iii) *Negative Factors.*—It must be clearly understood that some of the C's and some of the E's may stand for negative characteristics, *i.e.*, for the absence of certain positive characteristics. Negative conditions may be just as important as positive ones. *E.g.*, there is no general law about the effect of heat on oxygen. If the oxygen be free from contact with other gases it merely expands when heated. If it be mixed with a sufficient proportion of hydrogen it explodes. Thus the negative condition "in absence of hydrogen" is an essential factor when the effect to be considered is the expansion of oxygen.

5. Plurality of Causes and Effects.

(i) *Total Cause and Total Effect.*—Before we can discuss whether plurality of causes or of effects is logically possible we must define the notions of "total cause" and "total effect." The definition is as follows:—

"$C_1 \ldots C_n$ stands to $E_1 \ldots E_m$ in the relation of ***total cause to total effect***" means that "$C_1 \ldots C_n$ is a S.S.C. of $E_1 \ldots E_m$, and it is not a S.C. of any characteristic outside the set $E_1 \ldots E_m$." (Df. 6.)

It will be seen that this definition is equivalent to the conjunction of the following three propositions, one of which is affirmative and the other two negative:—

(*a*) Any occurrence of $C_1 \ldots C_n$ is also an occurrence of $E_1 \ldots E_m$.
(*b*) There is no selection of factors from $C_1 \ldots C_n$ such that every occurrence of it is also an occurrence of $E_1 \ldots E_m$.
(*c*) There is no factor outside $E_1 \ldots E_m$ such that every occurrence of $C_1 \ldots C_n$ is also an occurrence of it.

(ii) *Plurality of Causes.*—With this definition it is logically possible for ***several*** different sets of factors to stand to *one and the same* set of factors in the relation of total cause to total effect. For we have proved in Prop. 5 that one and the same E can have a plurality of different S.S.C.'s. We also showed there that the various S.S.C.'s may either have no factor in common or may partially overlap, but that one cannot be wholly included in another. We also showed in Prop. 6 that any factor which is common to all possible S.S.C.'s of a given E is a N.C. of that E. It is, of course, quite possible for an effect to have *no* necessary conditions. For if it has two S.S.C.'s which have no factor in common, it cannot possibly have a N.C. On the other hand (Prop. 7), if an effect has only one S.S.C. this is also the G.N.C. of the effect. So, when there is no plurality of causes, the total cause of a given total effect is a set of factors which are severally necessary and jointly sufficient to produce the effect.

Thus our definitions allow the possibility of a plurality of *total causes* for one and the same *total effect.* Whether there actually is such plurality in nature, or whether the appearance of it is always due to our partial ignorance or inadequate analysis, is a question into which I shall not enter here. Of course, even if a given total effect does have a plurality of total causes, *each particular occurrence* of this total effect will be determined by the occurrence of one and only one of these total causes. The plurality will show itself in the fact that some occurrences of the total effect will be determined by occurrences of one of the total causes, whilst other occurrences of the total effect will be determined by occurrences of another of the total causes.

(iii) *Plurality of Effects.*—It is plain from Df. 6 that a given total cause could not have more than one total effect. Thus plurality of total effects is ruled out by our definitions.

6. Formal Statement of Mill's Methods.—We are now in a position to deal with Mill's Methods of Agreement and Difference. Mill never clearly defined what he meant by "cause" or by "effect," and he never clearly stated what suppressed premises, if any, were needed by his Methods. We shall now be able to see exactly in what sense "cause" and "effect" are used in each application of each Method; what assumptions are tacitly made; and what bearing the question of "plurality of causes" has on the validity of each application of each Method. Mill made two applications of each Method, *viz.*, to find "the effect of a given cause" and to find "the cause of a given effect." We have therefore in all four cases to consider:

(i) *Method of Agreement.*—(*a*) *To find the "effect" of* A.

The premises are:—

All ABC is *abc*; and
All ADE is *ade*.

The argument should then run as follows:—

A is not a S.C. of *bc*; for in the second case A occurs without *bc*. It is assumed that A is a S.C. of *something* in *abc*. Therefore it must be a sufficient condition of *a*.

Thus, the suppressed premise is that A is a S.C. of something or other in *abc*. And the sense in which it is proved that the effect of A is *a* is that it is *a* of which A is a S.C.

(*b*) *To find the "cause" of a.*

The premises are as before.

The argument should run as follows:—

From the two premises it follows that both ABC and ADE are S.C.'s of *a*. But every S.C. of *a* must contain all the N.C.'s of *a*. (Prop. 3.)

Therefore, if *a* has a N.C. at all, it must be or be contained in the common part of the two S.C.'s of *a*.

But the only common part is A.

Therefore, if *a* has a N.C. at all, either A itself or some part of A must be a N.C. of *a*.

Thus the sense in which it is proved that the cause of *a* is A or some part of A is that if *a* has a N.C. at all then it is A or some part of A which is its N.C.

Mill's contention that, in this application, the Method of Agreement is rendered uncertain by the possibility of Plurality of Causes is true, and has the following meaning. If it be admitted that *a* may have more than one S.S.C. it is possible that it may have no N.C. at all. In fact, this will be the case if there is no factor common to all its S.S.C.'s. Thus, we cannot draw the categorical conclusion that the N.C. of *a* is or is contained in A unless we are given the additional premise: "*a* has either only one S.S.C., or, if it has several, there is a factor common to all of them."

(ii) *Method of Difference.*—(*a*) *To find the "effect" of* A.

The premises are :—

All ABC is *abc* ; and
All (non-A)BC is (non-*a*)*bc*.

The argument should run as follows :—

A is not a N.C. of *bc*, for in the second case *bc* occurs without A.

It is assumed that A is a N.C. of *something* in *abc*.

Therefore A must be a N.C. of *a*.

Thus the suppressed premise is that A is a N.C. of *something* in *abc*. And the sense in which it is proved that the effect of A is *a* is that it is *a* of which A is a N.C.

(*b*) *To find the "cause" of a.*

The premises are as before.

The argument should run as follows :—

It follows from the second premise that All (non-A)BC is non-*a*.

Therefore, by contraposition, All *a* is non-[(non-A)BC].

Therefore, All *a* is either A or non-(BC).

Therefore, All *a* which is BC is A.

This may be stated in the form: "In presence of BC, A is necessary to produce *a*."

Now, the first premise could be put in the form: "In presence of BC, A is sufficient to produce *a*."

Combining these, we reach the final conclusion: "In presence of BC, A is necessary and sufficient to produce *a*."

We have no right to conclude that A would be either necessary or sufficient in the absence of BC. In the presence of a suitable mixture of hydrogen and oxygen a spark is both necessary and sufficient to produce an explosion with the formation of water. But it is not sufficient in the absence of either of the two gases. Again, when a person is in good general health, prolonged and concentrated exposure to infection is necessary and sufficient to give him a cold. But when he is in bad general health it is not necessary that the exposure should be either prolonged or concentrated.

Thus Mill has no right to draw the unqualified conclusion that A is the cause of *a*, either in the sense of necessary or in the sense of sufficient condition. But he is justified in concluding that, in presence of BC, A is the cause of *a*, in the sense of being necessary and sufficient to produce *a*.

(iii) *The Joint Method.*—Mill's Joint Method is suggested as a method by which we may find the "cause" of *a* in cases where the Method of Difference cannot be used, and where the Method of Agreement is rendered untrustworthy by the possibility of Plurality of Causes.

It consists of two parts. The first is an ordinary application of the Method of Agreement. From this we reach the conclusion that, unless *a* has a plurality of S.S.C.'s with no factor common to all of them, A or some part of A is a N.C. of *a*. But, owing to the possibility of plurality of causes, it remains possible that A may be irrelevant to *a*. It may be, *e.g.*, that BC is a S.S.C. of *a* in the first case, and that DE is a S.S.C. of *a* in the second case, and therefore that *a* has no N.C. at all. The second part of the Joint Method is supposed to state conditions under which this possibility might be rejected. It is as follows. We are to look for a pair of instances which agree in *no* respect, positive or negative, except that A and *a* are absent from both of them. It is alleged by Mill that, if we find such a pair of instances, we can conclude with certainty that the "cause" of *a* is A.

It is, of course, quite plain that, even if the method were logically unimpeachable, it would be perfectly useless in practice. Any pair of instances that we could possibly find would agree in innumerable *negative* characteristics beside the absence of A and the absence of *a*. But is the argument logically sound even if premises of the required kind could be found?

It would run as follows. Since our two instances are to agree in *no* respect, positive or negative, except the absence of A and of *a*, BC cannot be absent in both of them. Therefore BC must be present in one of them. But *a* is absent in both of them. Therefore, in one of them BC is present without *a* being present. Therefore BC cannot be a S.C. of *a*. But, from the first part of the method, we know that ABC is a S.C. of *a*. A precisely similar argument would show that DE cannot be a S.C. of *a*. And, from the first part of the method, we know that ADE is a S.C. of *a*. Mill thinks that we can conclude that A is a N.C. of *a*. This, however, is a mistake. All that we can conclude is that, *in presence of BC or DE*, A is a N.C. of *a*. It remains quite possible that there is another S.S.C. of *a*, *e.g.*, XYZ, which does not contain A at all. And, in that case, A could not be a N.C., without qualification, of *a*. *E.g.*, a certain kind of soil, when treated with lime, always yields good crops; and, when lime is absent, good crops are absent on this soil. This proves that the presence of lime is a necessary condition for getting good crops *with this kind of soil*. But it does not prove that the presence of lime is a necessary condition, without qualification, for getting good crops. With other kinds of soil it might be unnecessary or positively harmful.

There is, however, a perfectly sensible method of argument, which is not Mill's, but which might fairly be called the *Joint Method*. The first part of it would be to take a large number of sets of characteristics, such that each set contains A and that in other respects they are as unlike each other as possible. One would try to arrange that A should be the only characteristic common to *all* of them, though it might be impossible to arrange that any *two* of them had only A in common. Suppose it were found that every occurrence of each of these sets was also an occurrence of *a*. Then there would be a strong presumption, though never a rigid proof, that A was a S.C. of *a*. The alternative would be that *a* had an enormous number of alternative S.S.C.'s. The second part of the method would be to take a large number of sets of characteristics, such that each set *lacks* A, and that in other respects they are as unlike each other as possible. One would try to arrange that non-A should be the only characteristic common to *all* of them, though it might be impossible to arrange that any *two* of them had only non-A in common. Suppose it were found that every occurrence of each of these sets was also characterised by the *absence* of *a*. Then there would be a strong presumption, though never a rigid proof, that non-A was a S.C. of non-*a*. It would then follow by contraposition that A was a N.C. of *a*. Thus the combination of the two sets of observations would make it probable that A is a necessary and sufficient condition of *a*. The argument is, of course, greatly strengthened if the characteristics other than A and *a* which occur among the sets of the first series are, as nearly as may be, the same as the characteristics other than non-A and non-*a* which occur among the sets of the second series. Thus, as Mr. Johnson has pointed out, the various sets of the same series should differ as much as possible in all respects except the one under investigation; whilst the two series, as wholes, should agree as much as possible in all respects except the one under investigation. A good example would be provided by the empirical arguments which lead to the conclusion that the property of having an asymmetrical molecular structure is a necessary and sufficient condition of the property of rotating the plane of polarisation of plane-polarised light. As Mill's own Joint Method is both useless and invalid, the name of "Joint Method" might be reserved in future for the above important and legitimate, though not absolutely conclusive, type of inductive argument.

(*To be concluded.*)